

# **SPARSE MIMO OFDM CHANNEL ESTIMATION AND PAPR REDUCTION USING GENERALIZED INVERSE TECHNIQUE**

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## **ABSTRACT**

*MIMO-OFDM systems provide high spectral efficiency for wireless communication systems. However, they have a major drawback of high PAPR which results in inefficient use of a power amplifier and also improper detections. Now a days, in wireless communication systems, channel estimation is mandatory for higher data rates with low bit error rates. For reducing burden on system the channel estimation results are exploited to reduce the high PAPR by using the technique called SVD based Generalized Inverse. From the results we can say that Sparse MIMO OFDM Channel Estimation Using Spatial and Temporal Correlations is the best for channel estimation.*

**Keywords:** *Generalized Inverse, MIMO-OFDM, PAPR, SVD.*

## **I. INTRODUCTION**

Multiple Input Multiple Output (MIMO)-OFDM is widely recognized as a key technology for future wire-less communications due to its high spectral efficiency and superior robustness to multipath fading channels [1]. For MIMO-OFDM systems, accurate channel estimation is essential to guarantee the system performance [2].

In this letter, a more practical sparse MIMO-OFDM channel estimation scheme based on spatial and temporal correlations of sparse wireless MIMO channels is proposed to deal with arbitrary path delays and also we exploit a generalized inverse of the right singular matrix of the MIMO channel to use redundant spatial dimensions at the transmitter. The generalized inverse of a matrix inherently includes an arbitrarily controllable matrix which is our key design parameter to minimize PAPR, and has a fixed part that we use for obtaining the spatial multiplexing gain.

The main contributions of this letter are summarized as follows. First, the proposed scheme can achieve super-resolution estimates of arbitrary path delays, which is more suitable for wireless channels in practice. Second, due to the small scale of the transmit and receive antenna arrays compared to the long signal transmission distance in typical MIMO antenna geometry, channel impulse responses (CIRs) of different transmit-receive antenna pairs share common path delays [5], which can be translated as a common sparse pattern of CIRs due to the spatial correlation of MIMO channels.

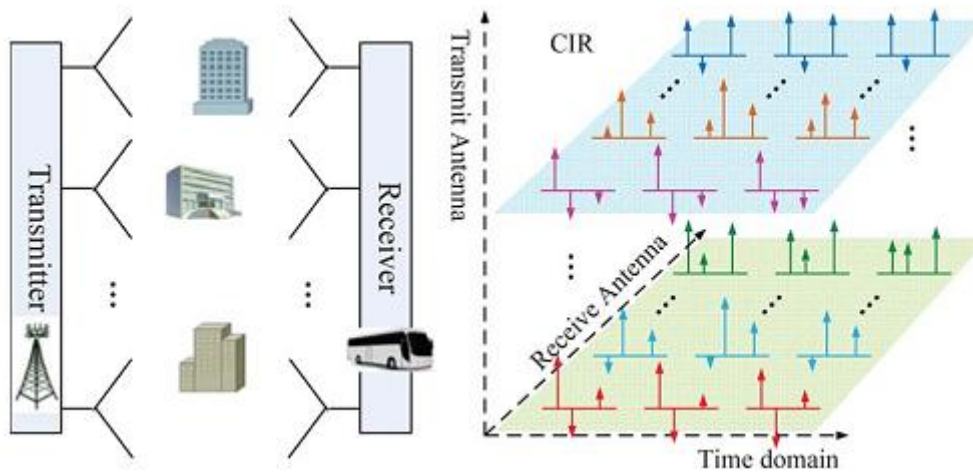
Meanwhile, such common sparse pattern is nearly unchanged along several adjacent OFDM symbols due to the temporal correlation of wireless channels [6], [7]. Compared with previous work which just simply extends the sparse channel estimation scheme in single antenna systems to that in MIMO by exploiting the spatial correlation of MIMO channels [5] or only considers the temporal correlation for single antenna systems [6], [7], the proposed scheme exploits both spatial and temporal correlations to improve the channel estimation accuracy. Third, we reduce the pilot overhead by using the finite rate of innovation (FRI) theory [8], which can recover the analog sparse signal with very low sampling rate, as a result, the average pilot overhead per antenna only depends on the channel sparsity level instead of the channel length. Finally, PAPR performance for large-scale MIMO systems, has effectively improved.

**II. SPARSE MIMO CHANNEL MODEL**

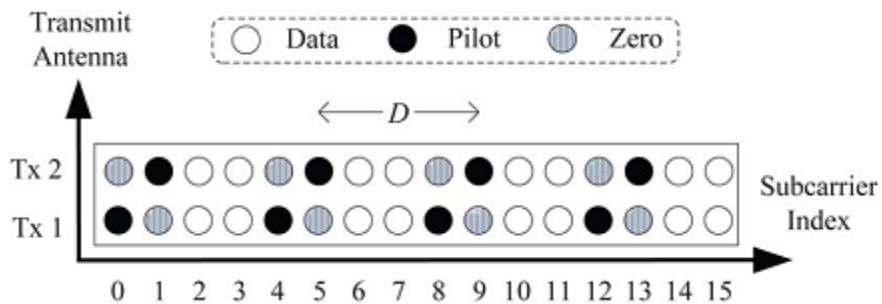
The MIMO channel is shown in Fig.1, and its following characteristics will be considered in this project.

1) Channel Sparsity: In typical outdoor communication scenarios, the CIR is intrinsically sparse due to several significant scatterers [3], [5]. For an  $N_t \times N_r$  MIMO system, the CIR  $h_{(i,j)}(t)$  between the  $i$ th transmit antenna and the  $j$ th receive antenna can be modeled as [1],

$$h_{(i,j)}(t) = \sum_{p=1}^P \alpha_p^{(i,j)} \delta(t - \tau_p^{(i,j)}), 1 \leq i \leq N_t, 1 \leq j \leq N_r \tag{1}$$



**Fig 1. Spatial and temporal correlations of wireless MIMO channels**



**Fig 2. Pilot pattern. Note that the specific  $N_t = 2, D = 4, N_p = 4,$  and  $N_{p\_total} = 8$  are used for illustration purpose.**

Where  $\delta(\cdot)$  is the Dirac function,  $P$  is the total number of resolvable propagation paths, and  $\tau_p^{(i,j)}$  and  $\alpha_p^{(i,j)}$  denote the path delay and path gain of the  $p^{\text{th}}$  path, respectively.

2) Spatial Correlation: Because the scale of the transmit or receive antenna array is very small compared to the long signal transmission distance, channels of different transmit-receive antenna pairs share very similar scatterers. Meanwhile, for most communication systems, the path delay difference from the similar scatterer is far less than the system sampling period. Therefore, CIRs of different transmit-receive antenna pairs share a common sparse pattern, although the corresponding path gains may be quite different [5].

3) Temporal Correlation: For wireless channels, the path delays vary much slowly than the path gains, and the path gains vary continuously [6]. Thus, the channel sparse pattern is nearly unchanged during several adjacent OFDM symbols, and the path gains are also correlated [7].

### III. SPARSE MIMO-OFDM CHANNEL ESTIMATION

In this section, the widely used pilot pattern is briefly introduced at first, based on which a super-resolution sparse MIMO OFDM channel estimation method is then applied. Finally, the required number of pilots is discussed under the framework of the FRI theory.

#### 3.1 Pilot Pattern

The pilot pattern widely used in common MIMO-OFDM systems is illustrated in Fig. 2. In the frequency domain,  $N_p$  pilots are uniformly spaced with the pilot interval  $D$  (e.g.,  $D = 4$  in Fig. 2). Meanwhile, every pilot is allocated with a pilot index  $l$  for  $0 \leq l \leq N_p - 1$ , which is ascending with the increase of the subcarrier index. Furthermore, to distinguish MIMO channels associated with different transmit antennas, each transmit antenna uses a unique subcarrier index initial phase  $\theta_i$  for  $1 \leq i \leq N_t$  and  $(N_t - 1)N_p$  zero subcarriers to ensure the orthogonality of pilots [4]. Therefore, for the  $i^{\text{th}}$  transmit antenna, the subcarrier index of the  $l^{\text{th}}$  pilot is

$$I_{\text{pilot}}^i(l) = \theta_i + lD, 0 \leq l \leq N_p - 1 \tag{2}$$

Consequently, the total pilot overhead per transmit antenna is  $N_{p\_total} = N_t N_p$ , and thus,  $N_p$  can be also referred as the average pilot overhead per transmit antenna in this paper.

#### 3.2 Super-Resolution Channel Estimation

At the receiver, the equivalent baseband channel frequency response (CFR)  $H(f)$  can be expressed as

$$H(f) = \sum_{p=1}^P \alpha_p e^{-j2\pi f \tau_p}, -f_s/2 \leq f \leq f_s/2 \tag{3}$$

Where superscripts  $i$  and  $j$  in (1) are omitted for convenience,  $f_s = 1/T_s$  is the system bandwidth, and  $T_s$  is the sampling period. Meanwhile, the  $N$ -point discrete Fourier transform (DFT) of the time-domain equivalent baseband channel can be expressed as [5], i.e.,

$$H[k] = H\left(\frac{kf_s}{N}\right), 0 \leq k \leq N - 1 \tag{4}$$

Therefore, for the  $(i,j)$ th transmit-receive antenna pair, according to (2)–(4), the estimated CFRs over pilots can be written as

$$\hat{H}^{(i,j)}[l] = H[I_{\text{pilot}}^i(l)] = H\left(\frac{(\theta_i + lD)f_s}{N}\right)$$

$$= \sum_{p=1}^P \alpha_p^{(i,j)} e^{-j2\pi \frac{(\hat{\theta}_i+1D)f_s \tau_p^{(i,j)}}{N}} + W^{(i,j)} [l] \tag{5}$$

Where  $\hat{H}^{(i,j)} [l]$  for  $0 \leq l \leq N_p - 1$  can be obtained by using the conventional minimum mean square error (MMSE) or least square (LS) method [2], and  $W^{(i,j)} [l]$  is the additive white Gaussian noise (AWGN). Eq. (4.5) can be also written in a vector form as

$$\hat{H}^{(i,j)} [l] = (\mathbf{v}^{(i,j)} [l])^T \mathbf{a}^{(i,j)} + W^{(i,j)} [l] \tag{6}$$

Because the wireless channel is inherently sparse and the small scale of multiple transmit or receive antennas is negligible compared to the long signal transmission distance, CIRs of different transmit-receive antenna pairs share common path delays, which is equivalently translated as a common sparse pattern of CIRs due to the spatial correlation of MIMO channels [5], i.e

$$\tau_p^{(i,j)} = \tau_p \text{ and}$$

$$\mathbf{V}^{(i,j)} [l] = \mathbf{v} [l] \text{ for } 1 \leq p \leq P, 1 \leq i \leq N_t, 1 \leq j \leq N_r$$

Hence, by exploiting such spatially common sparse pattern shared among different receive antennas associated with the  $i^{th}$  transmit antenna, we have

$$\hat{H}^i = \mathbf{V} \mathbf{A}^i + \mathbf{W}^i, 1 \leq i \leq N_t \tag{7}$$

Where the  $N_p \times N_r$  measurement matrix  $\hat{H}^i$  is

$$\hat{H}^i = \begin{bmatrix} \hat{H}^{(i,1)} [0] & \hat{H}^{(i,2)} [0] & \dots & \hat{H}^{(i,N_r)} [0] \\ \hat{H}^{(i,1)} [1] & \hat{H}^{(i,2)} [1] & \dots & \hat{H}^{(i,N_r)} [1] \\ \hat{H}^{(i,1)} [N_p - 1] & \hat{H}^{(i,2)} [N_p - 1] & \dots & \hat{H}^{(i,N_r)} [N_p - 1] \end{bmatrix}$$

When all  $N_t$  transmit antennas are considered based on (7), we have

$$\hat{H} = \mathbf{V} \mathbf{A} + \mathbf{W} \tag{8}$$

Comparing the formulated problem (8) with the classical direction-of-arrival (DOA) problem [9], we find out that they are mathematically equivalent. Specifically, the traditional DOA problem is to typically estimate the DOAs of the  $P$  sources from a set of time-domain measurements, which are obtained from the  $N_p$  sensors outputs at  $N_t N_r$  distinct time instants (time-domain samples). In contrast to our problem in (8), we try to estimate the path delays of  $P$  multipath from a set of frequency-domain measurements, which are acquired from  $N_p$  pilots of  $N_t N_r$  distinct antenna pairs (antenna-domain samples). It has been verified in [10] that the total least square estimating signal parameters via rotational invariance techniques (TLS-ESPRIT) algorithm in [9] can be applied to (8) to efficiently estimate path delays with arbitrary values.

By using the TLS-ESPRIT algorithm, we can obtain super resolution estimates of path delays, i.e.,  $\hat{\tau}_p$ , for  $1 \leq p \leq P$ , and thus,  $\hat{V}$  can be obtained accordingly. Then, path gains can be acquired by the LS method [7],

$$\text{i.e. } \hat{A} = \hat{V} + \hat{H} = (\hat{V}^H \hat{V})^{-1} \hat{V}^H \hat{H} \tag{9}$$

For a certain entry of  $\hat{A}$  i.e.,  $\hat{\alpha}_p^{(i,j)} \gamma^{\theta_i \hat{\tau}_p}$  because  $\theta_i$  is known at the receiver and  $\hat{\tau}_p$  has been estimated after applying TLS-ESPRIT algorithm, we can easily obtain the estimation of the path gain  $\hat{\alpha}_p^{(i,j)}$  for  $1 \leq p \leq P, 1 \leq i \leq N_t, 1 \leq j \leq N_r$ . Finally, the complete CFR estimation over all OFDM subcarriers can be obtained based on (3) and (4).

Furthermore, we can also exploit the temporal correlation of wireless channels to improve the accuracy of the channel estimation. First, path delays of CIRs during several adjacent OFDM symbols are nearly unchanged [6], [7], which is equivalently referred as a common sparse pattern of CIRs due to the temporal correlation of MIMO channels. Thus, the Vandermonde matrix  $V$  in (8) remains unchanged across several adjacent OFDM symbols. Moreover, path gains during adjacent OFDM symbols are also correlated owing to the temporal continuity of the CIR, so as in (8) for several adjacent OFDM symbols are also correlated. Therefore, when estimating CIRs of the  $q^{\text{th}}$  OFDM symbol, we can jointly exploit  $\widehat{H}_q$  of several adjacent OFDM symbols based on (8), i.e.,

$$\frac{\sum_{p=q-R}^{q+R} \widehat{H}_p}{2R+1} = V_q \frac{\sum_{p=q-R}^{q+R} A_p}{2R+1} + \frac{\sum_{p=q-R}^{q+R} W_p}{2R+1} \quad (10)$$

Where the subscript  $p$  is used to denote the index of the OFDM symbol, and the common sparse pattern of CIRs is assumed in  $2R + 1$  adjacent OFDM symbols [7]. In this way, the effective noise can be reduced, so the improved channel estimation accuracy is expected.

In contrast to the existing nonparametric scheme which estimates the channel by interpolating or predicting based on CFRs over pilots [1], [2], our proposed scheme exploits the sparsity as well as the spatial and temporal correlations of wireless MIMO channels to first acquire estimations of channel parameters, including path delays and gains, and then obtain the estimation of CFR according to (3) and (4).

### 3.3 Discussion on Pilot Overhead

Compared with the model of the multiple filters bank based on the FRI theory [10], it can be found out that CIRs of  $N_t N_r$  transmit-receive antenna pairs are equivalent to the  $N_t N_r$  semiperiod sparse subspaces, and the  $N_p$  pilots are equivalent to the  $N_p$  multichannel filters. Therefore, by using the FRI theory, the smallest required number of pilots for each transmit antenna is  $N_p = 2P$  in a noiseless scenario. For practical channels with the maximum delay spread  $\tau_{\max}$ , although the normalized channel length  $L = \tau_{\max}/T_s$  is usually very large, the sparsity level  $P$  is small, i.e., PL [3]. Consequently, in contrast to the nonparametric channel estimation method where the required number of pilots heavily depends on  $L$ , our proposed parametric scheme only needs  $2P$  pilots in theory. Note that the number of pilots in practice is larger than  $2P$  to improve the accuracy of the channel estimation due to AWGN.

## IV. SYSTEM MODEL DESCRIPTION

As shown in Fig. 3, we consider a downlink single-user (SU) MIMO-OFDM system that consists of a transmitter equipped with  $M_T$  antennas and a receiver equipped with  $M_R$  antennas, where  $M_T > M_R \geq d_k$ . We assume the receiver perfectly reports channel information through an ideal feedback channel. The subscript  $k$  means the  $k^{\text{th}}$  subcarrier,  $\forall k \in \{1, \dots, N_C\}$ . The transmitter sends a  $d_k \times 1$  symbol vector  $\mathbf{s}_k = [s_{k,1}, \dots, s_{k,d_k}]^T$  then the received signal can be described as

$$\mathbf{y}_k = \mathbf{R}_k \mathbf{H}_k \mathbf{F}_k \mathbf{s}_k + \mathbf{R}_k \mathbf{n}_k \quad (11)$$

where  $\mathbf{F}_k$  denotes the transmission precoder for the  $k^{\text{th}}$  subcarrier.  $\mathbf{R}_k$  is the receiver filter of the  $k^{\text{th}}$  subcarrier, and the complex Gaussian noise vector  $\mathbf{n}_k$ .  $\mathbf{H}_k$  is a  $M_R \times M_T$  Rayleigh fading MIMO channel, and the frequency selective fading MIMO-OFDM signaling is assumed as a series of narrowband frequency flat fading signalings. For  $N_C$  subcarriers, the overall received signal can be denoted as

$$y = RHF_s + Rn \tag{12}$$

$$R_k = (H_k F_k)^{-1} \tag{13}$$

where  $\mathbf{n} = [n^T_1, \dots, n^T_{N_C}]^T$ ,  $\mathbf{s} = [s^T_1, \dots, s^T_{N_C}]^T$ ,  $\mathbf{F} = \text{blkdiag}(\mathbf{F}_1, \dots, \mathbf{F}_{N_C})$ ,  $\mathbf{R} = \text{blkdiag}(\mathbf{R}_1, \dots, \mathbf{R}_{N_C})$ , and  $\mathbf{H} = \text{blkdiag}(\mathbf{H}_1, \dots, \mathbf{H}_{N_C})$ . For finding  $R_k$  we have to apply Generalized inverse.

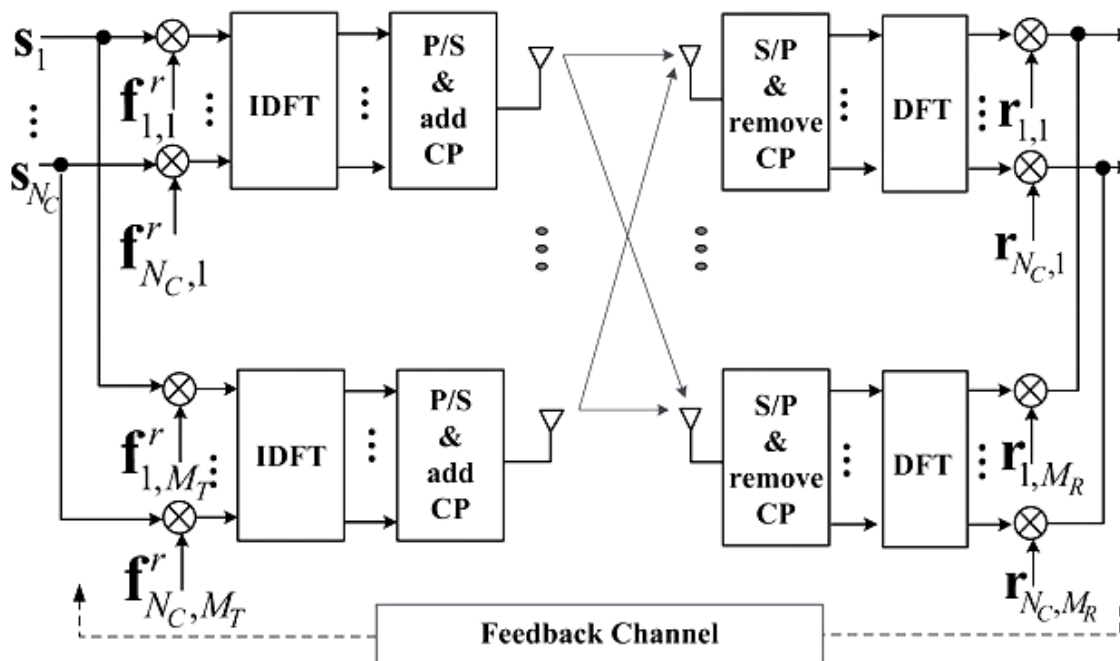


Fig. 3. System model of single user MIMO-OFDM system.

### V. ALGORITHM FOR THE GENERALIZED INVERSE OF A MATRIX

An algorithm for finding the generalized inverse of a matrix is as follows, according to Adetunde et al;

Step 1: In A of rank r, find any non-singular minor of order r call it M

Step 2: Invert M and transpose the inverse (M)

Step3: In A replace each element of M by the corresponding element of (M)

That is a = M the (s,t) element of m, then replace a b M, the (t,s) element of M equivalent to the (s,t) element of the transpose of M

Step4: Replace all the other elements of A by zero

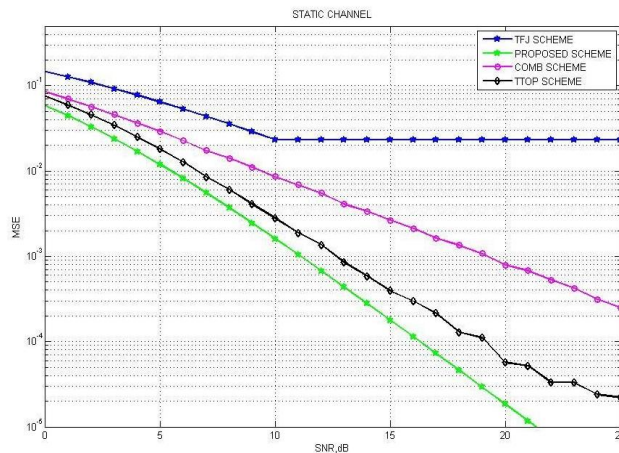
Step 5: Transpose the resulting matrix and the result is G a generalized inverse of A

### VI. SIMULATION RESULTS

A simulation study was carried out to compare the performance of the proposed scheme with those of the existing state-of-the-art methods for MIMO-OFDM systems. The conventional comb-type pilot and time-domain training based orthogonal pilot (TTOP) [2] schemes were selected as the typical examples of the nonparametric channel estimation scheme, while the recent time-frequency joint (TFJ) channel estimation scheme [4] was selected as an example of the conventional parametric scheme. System parameters were set as follows: the carrier frequency is  $f_c = 1$  GHz, the system bandwidth is  $f_s = 10$  MHz, the size of the OFDM

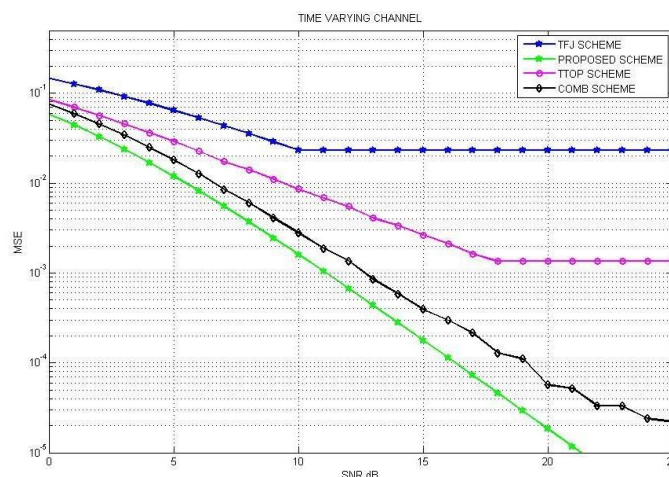


symbol is  $N = 4096$ , and  $N_g = 256$  is the guard interval length  $P$ , which can combat channels whose maximum delay spread is  $25.6 \mu s$ . The International Telecommunication Union Vehicular B (ITU-VB) channel model with the maximum delay spread  $20 \mu s$  and the number of paths  $P = 6$  [4] were considered.

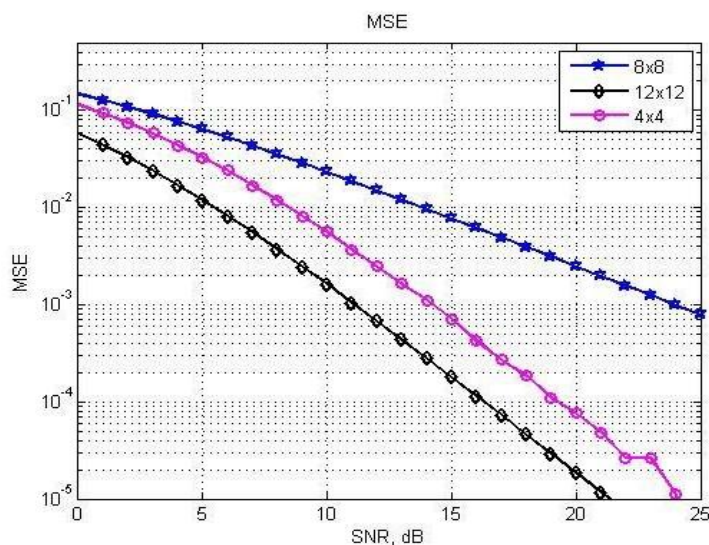


**Fig. 4. MSE performance comparison of different schemes in a  $4 \times 4$  MIMO System Static channel**

Fig.4 & Fig.5 compares the mean square error (MSE) performance of different channel estimation schemes. Both the static ITUVB channel and the time-varying ITU-VB channel with the mobile speed of 90 km/h in a  $4 \times 4$  MIMO system were considered. The comb-type pilot based scheme used  $N_p = 256$  pilots, the TTOP scheme used  $N_p = 64$  pilots with  $T$  adjacent OFDM symbols for training, where  $T = 4$  for the time-varying channel and  $T = 8$  for the static channel to achieve better performance, the TFJ scheme adopted time-domain training sequences of 256-length and  $N_p = 64$  pilots, and our proposed scheme used  $N_p = 64$  pilots with  $R = 4$  for fair comparison. Moreover, for the time-varying ITU-VB channel, the superior performance of our proposed parametric scheme to conventional nonparametric schemes is more obvious.

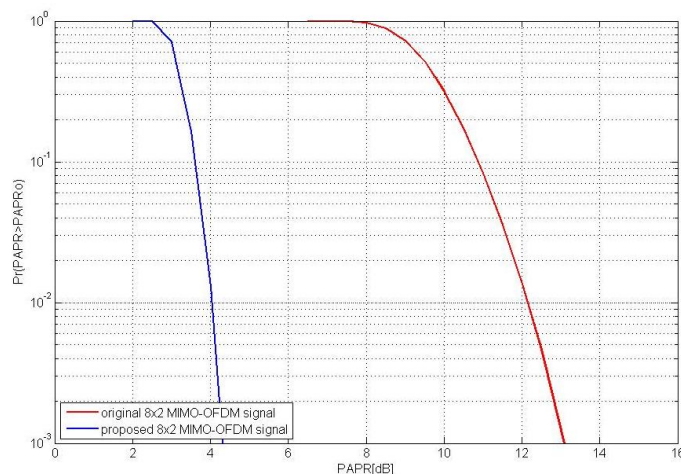


**Fig. 5. MSE performance comparison of different schemes in a  $4 \times 4$  MIMO System time-varying channel with the mobile speed of 90km/h**



**Fig. 6. MSE performance of the proposed scheme in  $4 \times 4$ ,  $8 \times 8$ , and  $12 \times 12$  MIMO systems.**

The MSE performance of the proposed scheme in  $12 \times 12$  MIMO system is superior to that in  $8 \times 8$  MIMO system by 5 dB with the same  $N_p$  and outperforms that in  $4 \times 4$  MIMO system with the reduced  $N_p$ . Fig.6, reveals that with the increased number of antennas, the MSE performance improves with the same  $N_p$ . Equivalently, to achieve the same channel estimation accuracy; the required number of pilots  $N_p$  can be reduced. Fig. 7 shows the reduction of PAPR with SVD based Generalized inverse compare to the original MIMO-OFDM technique.



**Fig. 7. Comparison of reduction in PAPR between original and propose schemes**

## VII. CONCLUSION

The proposed super-resolution sparse MIMO channel estimation scheme exploits the sparsity as well as the spatial and temporal correlations of wireless MIMO channels. super-resolution estimates of path delays with arbitrary values can achieve higher channel estimation accuracy than conventional schemes. Under the framework of the FRI theory, the required number of pilots in the proposed scheme is obviously less than that in nonparametric channel estimation schemes. Moreover, simulations demonstrate that the average pilot overhead per transmit antenna will be interestingly reduced with the increased number of antennas. It is observed that the



PAPR value of proposed scheme (3.8dB) is decreasing with same number of transmitting and receiving antennas compare to the original scheme (13dB).

## REFERENCES

- [1] G. Stuber et al., "Broadband MIMO-OFDM wireless communications," Proc. IEEE, vol. 92, no. 2, pp. 271–294, Feb. 2004.
- [2] I. Barhumi, G. Leus, and M. Moonen, "Optimal training design for MIMO OFDM systems in mobile wireless channels," IEEE Trans. Signal Process., vol. 3, no. 6, pp. 958–974, Dec. 2009.
- [3] W. U. Bajwa, J. Haupt, A. M. Sayeed, and R. Nowak, "Compressed channel sensing: A new approach to estimating sparse multipath channels," Proc. IEEE, vol. 98, no. 6, pp. 1058–1076, Jun. 2010.
- [4] L. Dai, Z. Wang, and Z. Yang, "Spectrally efficient time-frequency training OFDM for mobile large-scale MIMO systems," IEEE J. Sel. Areas Commun., vol. 31, no. 2, pp. 251–263, Feb. 2013.
- [5] Y. Barbotin and M. Vetterli, "Estimation of sparse MIMO channels with common support," IEEE Trans. Commun., vol. 60, no. 12, pp. 3705–3716, Dec. 2012.
- [6] I. Telatar and D. Tse, "Capacity and mutual information of wideband multipath fading channels," IEEE Trans. Inf. Theory, vol. 46, no. 4, pp. 1384–1400, Jul. 2000.
- [7] L. Dai, J. Wang, Z. Wang, P. Tsiaflakis, and M. Moonen, "Spectrum and energy-efficient OFDM based on simultaneous multi-channel reconstruction," IEEE Trans. Signal Process., vol. 61, no. 23, pp. 6047–6059, Dec. 2013.
- [8] P. L. Dragotti, M. Vetterli, and T. Blu, "Sampling moments and reconstructing signals of finite rate of innovation: Shannon meets Strang-Fix," IEEE Trans. Signal Process., vol. 55, no. 5, pp. 1741–1757, May 2007.
- [9] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," IEEE Trans. Acoust., Speech, Signal Process., vol. 37, no. 7, pp. 984–995, Jul. 1989.
- [10] K. Gedlyahu and Y. C. Eldar, "Time-delay estimation from low-rate samples: A union of subspaces approach," IEEE Trans. Signal Process., vol. 58, no. 6, pp. 3017–3031, Sep. 2010.
- [11] Hyun-Su Cha, Hyukjin Chae, "Generalized Inverse Aided PAPR-Aware Linear Precoder Design for MIMO-OFDM System," IEEE communications letters. Vol 18. NO. 8. AUGUST 2014