

# REVIEW OF KINEMATIC MODELLING AND CONTROL OF 5 DEGREE OF FREEDOM ROBOTIC ARM USING D-H REPRESENTATION

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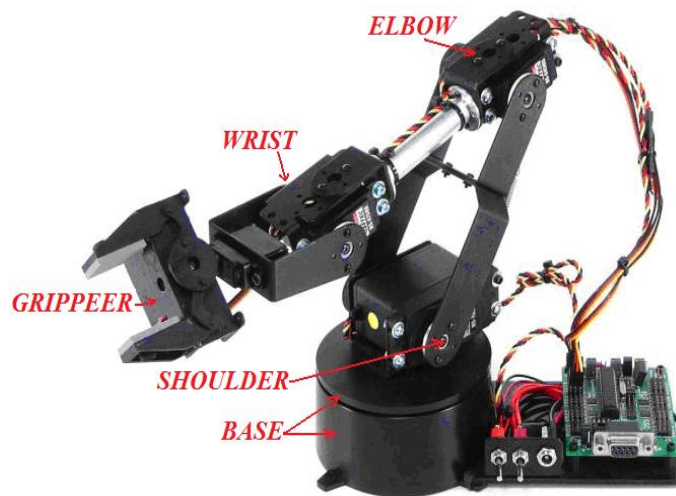
## ABSTRACT

*In this paper modeling and control of 5 degree of freedom robotic arm is the presented. It consists of forward and inverse kinematics is derived based on Denavit-Hartenberg(DH)representation. The main objective of this paper is to model the robotic arm by using D-H parameters. The kinematics problem is defined as the transformation from the Cartesian space to the joint space and vice versa. This paper aims to model the forward and inverse kinematics of a 5 DOF Robotic Arm for simple pick and place application. A general D-H representation of forward and inverse matrix is obtained.*

**Keywords:** *Forward Kinematics, Inverse Kinematics, Robotic Arm, Degrees of Freedom (DOF), Denavit-Hartenberg Representation*

## I. INTRODUCTION

In the recent years, robotics research has been aimed at finding solutions to the technical necessities of applied robotics. In the early 1960s, the industrial revolution put industrial robots in the factory to release the human operator from risky and harmful tasks. The later incorporation of industrial robots into other types of production processes added new requirements that called for more flexibility and intelligence in industrial robots. Currently, the creation of new needs and markets outside the traditional manufacturing robotic market and the aging world we live in is demanding field and service robots to attend to the new. In this paper the modeled robot manipulator arm is 5 DOF robotic arm. The number of joints present in the arm indicates the Degrees of Freedom. Here 5 DOF means robot arm contains 5 joints. These joints are of two types revolute or rotatory (Human joints) and prismatic or sliding joints. The kinematics solution of any robot manipulator consists of two sub problems forward and inverse kinematics. market and to human social needs.



**Figure 1.1 AL5B Robot arm [17]**

Forward kinematics will determine where the robot's arm position will be if all joints are known whereas inverse kinematics will calculate what each joint variable must be if the desired position and orientation of end-effector is determined. Hence Forward kinematics is defined as transformation from joint space to Cartesian space whereas Inverse kinematics is defined as transformation from Cartesian space to joint space. The forward and inverse kinematics of robot arm manipulator are derived based on Denavit-Harterberg (DH) Representation.

## **II. DEGREE OF FREEDOM**

The DOF, is a very important term to understand. Each degree of freedom is a joint on the arm, a place where it can bend or rotate or translate. You can typically identify the number of degrees of freedom by the number of actuators on the robot arm. When building a robot arm few degrees of freedom is allowed for the application, because each degree requires a motor, often an encoder, and exponentially complicated algorithms and cost [24]. The number of joints present in the arm indicates the Degrees of Freedom.

### **2.1 Types of joints**

There are mainly four types of joints that are found in robot manipulators:

- Revolute, rotary or pin joint
- Prismatic or sliding joint
- Spherical or ball joint
- Helical or screw joint

The revolute joint allows a rotation between the two connecting links. The best example of this is the hinge used to attach a door to the frame. The prismatic joint allows a pure translation between the two connecting links. The connection between a piston and a cylinder in an internal combustion engine or a compressor is via a prismatic joint. The spherical joint between two links allows the first link to rotate in all possible ways with respect to the second. The best example of this is seen in the human body. The shoulder and hip joints, called ball and socket joints, are spherical joints. The helical joint allows a helical motion between the two connecting bodies. A good example of this is the relative motion between a bolt and a nut.

### III. KINEMATICS

It is the branch of classical mechanics that describes the motion of bodies (objects) and systems (groups of objects) without consideration of the forces that cause the motion. Kinematics is the process of calculating the position in space of the end of a linked structure, given the angles of all the joints. This process can be extremely useful in robotics. You may have a robotic arm which needs to grab an object. If the software knows where the object is in relation to the shoulder, it simply needs to calculate the angles of the joints to reach it. The simplest application of kinematics is for particle motion, translational or rotational [8]. The next level of complexity comes from the introduction of rigid bodies, which are collections of particles having time invariant distances between themselves. Rigid bodies might undergo translation and rotation or a combination of both. A more complicated case is the kinematics of a system of rigid bodies, which may be linked together by mechanical joints. It is of two types [19].

- Forward Kinematics
- Inverse Kinematics

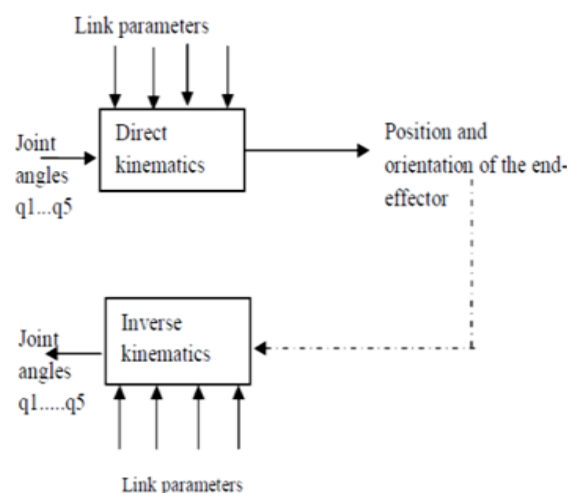


Fig.2 Forward and Inverse kinematics model [19]

#### 3.1 Forward Kinematics

The essential concept of forward kinematic animation is that the positions of particular parts of the model at a specified time are calculated from the position and orientation of the object, together with any information on the joints of an articulated model. So for example if the object to be animated is an arm with the shoulder remaining at a fixed location, the location of the tip of the thumb would be calculated from the angles of the shoulder, elbow, wrist, of the tip of the thumb would be calculated from the angles of the shoulder, elbow, wrist, thumb and knuckle joints. Three of these joints (the shoulder, wrist and the base of the thumb) have more than one degree of freedom, all of which must be taken into account. If the model were an entire human figure, then the location of the shoulder would also have to be calculated from other properties of model [20]. A manipulator is composed of serial links which are affixed to each other revolute or prismatic joints from the base frame through the end-effector. Calculating the position and orientation of the end-effector in terms of the joint variables is called as forward kinematics. In order to have forward kinematics for a robot mechanism in a systematic manner, one should use a suitable kinematics model.

### 3.2 Inverse Kinematics

It will enable us to calculate what each joint variable must be if we desire that the hand be located at particular point and have a particular position. The position and orientation of the end effector relative to the base frame compute all possible sets of joint angles and link geometries which could be used to attain the given position and orientation of the end effector [20]. Inverse Kinematics (IK) analysis determines the joint angles for desired position and orientation in Cartesian space. Inverse Kinematics is more difficult problem than forward kinematics. The solution of inverse kinematic is more complex than direct kinematics. The inverse kinematics problem has a wide range of applications in robotics. Most of our high level problem solving about the physical world is posed in Cartesian space. While we can reason about the physical world in Cartesian terms, the robot is actuated in joint space - that is what we ultimately can control. Once we solve a problem for its Cartesian space constraints, we need to map these constraints into the robot's joint space using inverse kinematics. For example, if we specify a straight line trajectory for a robot arm, we need to break that trajectory into a set of joint space values over time to get the robot to follow the line.

## IV. CO-ORDINATE FRAME OF 5 DOF ROBOTIC ARM

Coordinate frames for the AL5B robotic arm are assigned as shown in Fig. 3. They are established using the principles of the Denavit-Hartenberg (D-H) convention. For the kinematic model of 5 dof robotic arm first we have to assign frame to each link starting from base (frame 0) to end-effector (frame 5).

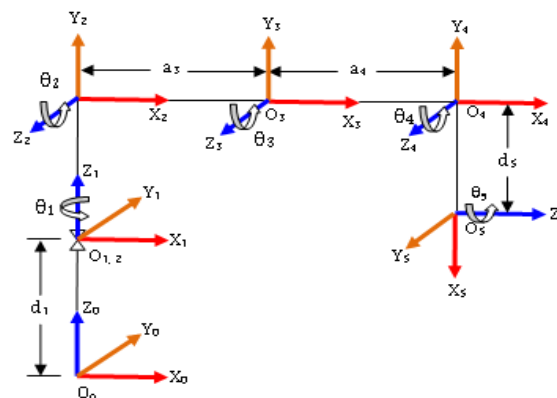


Fig. 3 Coordinate frame assignment [19]

## V. MATHEMATICAL AND KINEMATIC MODELING OF 5 DOF ROBOTIC ARM

Robot manipulator is named according to number of DOF, which refers to the number of joints. The robot manipulator arm has 5 joints, which mean the robot has 5DOF. The kinematics robot manipulator is derived by using Denavit-Hartenberg (DH) representation. In this convention, each homogeneous transformation  $A_i$  is represented as a product of four basic transformations.

$$T_e = R_z(\theta_i) D_z(d_i) D_x(a_i) R_x(\alpha_i) \quad (1)$$

$$\begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 & 1 & 0 & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

where  $R_x$  and  $R_z$  present rotation,  $D_x$  and  $D_z$  denote translation, and  $C\theta_i$  and  $S\theta_i$  are the short hands of  $\cos\theta_i$  and  $\sin\theta_i$ , respectively. The forward kinematics of the end-effector with respect to the base frame is determined by multiplying all of the  ${}^0T_5$  matrices. The four quantities  $\theta_i$ ,  $a_i$ ,  $d_i$ ,  $\alpha_i$  are parameters associated with link  $i$  and joint  $i$ . The four parameters  $a_i$ ,  $\alpha_i$ ,  $d_i$ , and  $\theta_i$  are generally given the names link length, link twist, link offset, and joint angle respectively.

An alternative representation of  $T_e$  can be written as:

$$T_e = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

where  $r_{kj}$ 's represent the rotational elements of transformation matrix ( $k$  and  $j=1, 2$  and  $3$ ).  $p_x$ ,  $p_y$  and  $p_z$  denote the elements of the position vector. For a five robotic arm, the position and orientation of the end-effector with respect

to the base is given by

$${}^0T_5 = {}^0T_1 {}^1T_2 {}^2T_3 {}^3T_4 {}^4T_5 \quad (4)$$

$${}^0T_1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3 * C_3 \\ S_3 & C_3 & 0 & a_3 * S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3T_4 = \begin{bmatrix} C_4 & -S_4 & 0 & a_4 * C_4 \\ S_4 & C_4 & 0 & a_4 * S_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4T_5 = \begin{bmatrix} C_5 & 0 & 0 & 0 \\ S_5 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_5 = {}^0T_1 {}^1T_2 {}^2T_3 {}^3T_4 {}^4T_5$$

$${}^0T_5 = \begin{bmatrix} C_{12}C_{345} & S_{12} & C_{12}S_{345} & S_{12}d_5 + C_{12}a_4C_{34} + C_{12}a_3C_3 \\ S_{12}C_{345} & -C_{12} & S_{12}S_{345} & -C_{12}d_5 + S_{12}a_4C_{34} + S_{12}a_3C_3 \\ S_{345} & 0 & -C_{345} & a_4S_{34} + a_5S_5 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_e = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$${}^0T_5 = T_e$$

Where,

$$r_{11} = C_{12}C_{345}$$

$$r_{12} = S_{12}$$

$$r_{13} = C_{12}S_{345}$$

$$r_{21} = S_{12}C_{345}$$

$$r_{22} = -C_{12}$$

$$r_{23} = S_{12}S_{345}$$

$$r_{31} = S_{345}$$

$$r_{32} = 0$$

$$r_{33} = -C_{345}$$

$$p_x = S_{12}d_5 + C_{12}a_4C_{34} + C_{12}a_3C_3$$

$$p_y = -C_{12}d_5 + S_{12}a_4C_{34} + S_{12}a_3C_3$$

$$p_z = a_4S_{34} + a_5S_5 + d_1$$

#### INVERSE KINEMATICS ANALYSIS

The following equations will be used to obtain the solution for the inverse Kinematics problem.

$${}^0T_5 = {}^0T_1 {}^1T_2 {}^2T_3 {}^3T_4 {}^4T_5 = T_e$$

Inverse kinematics solution for the first joint as a function of the known elements of  $T_e$ , the link transformation inverses are premultiplied as follows:

$$X_1 = [({}^0T_1)^{-1}] * {}^0T_5 = {}^1T_2 {}^2T_3 {}^3T_4 {}^4T_5 = {}^1T_5$$

Similarly, to find the other variables, the following equations are obtained in similar manner.

$$X_2 = [({}^0T_1)({}^1T_2)]^{-1} * {}^0T_5 = {}^2T_3 {}^3T_4 {}^4T_5 = {}^2T_5$$

$$X_3 = [({}^0T_1)({}^1T_2)({}^2T_3)]^{-1} * {}^0T_5 = {}^3T_4 {}^4T_5 = {}^3T_5$$

$$X_4 = [({}^0T_1)({}^1T_2)({}^2T_3)({}^3T_4)]^{-1} {}^0T_5 = {}^4T_5 = {}^4T_5$$

By solving these equations, we can calculate the values of  $\theta_1, \theta_2, \theta_3, \theta_4$  and  $\theta_5$

So, the values will be

$$\theta_1 = \theta_{12} - \theta_{12} \quad (6)$$

$$\theta_2 = \tan^{-1}(p_y/p_x) \quad (7)$$

$$\theta_3 = \tan^{-1}(S_3/C_3) \quad (8)$$

$$\theta_4 = \tan^{-1}(S_4/C_4) \pm \sqrt{1 + C_4^2} \quad (9)$$

$$\theta_5 = \theta_{345} - \theta_{34} \quad (10)$$

Using these value the values of all the joint angles i.e  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$  of 5 DOF robotic arm will be determined.

This is solution of denavit-hartenberg representation.

## VI. CONCLUSION

In this paper, complete analytical solution to the forward and inverse kinematics of 5 DOF Robotic arm is discussed. The forward kinematic analysis of 5 DOF robotic arm is investigated. A strategy based on geometric projection was done to resolve the inverse kinematics of 5 DOF robotic arm. A review from various kinematic modeling methods has been taken using denavit-hartenberg representation.

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